

Proof of Pascal's Rule

Pascal's Rule: $\binom{n+1}{k+1} = \binom{n}{k} + \binom{n}{k+1}$

Proof:
$$\begin{aligned}\binom{n+1}{k+1} &= \frac{(n+1)!}{(k+1)!(n+1-k-1)!} \\ &= \frac{(n+1)n!}{(k+1)!(n-k)!} \\ &= \frac{n!(n+1+k-k)}{(k+1)!(n-k)!} \\ &= \frac{n!(k+1) + n!(n-k)}{(k+1)!(n-k)!} \\ &= \frac{n!(k+1)}{(k+1)!(n-k)!} + \frac{n!(n-k)}{(k+1)!(n-k)!} \\ &= \frac{n!}{k!(n-k)!} + \frac{n!}{(k+1)!(n-k-1)!} \\ &= \binom{n}{k} + \binom{n}{k+1}\end{aligned}$$